Rate and State Friction and the Modeling of Aseismic Slip

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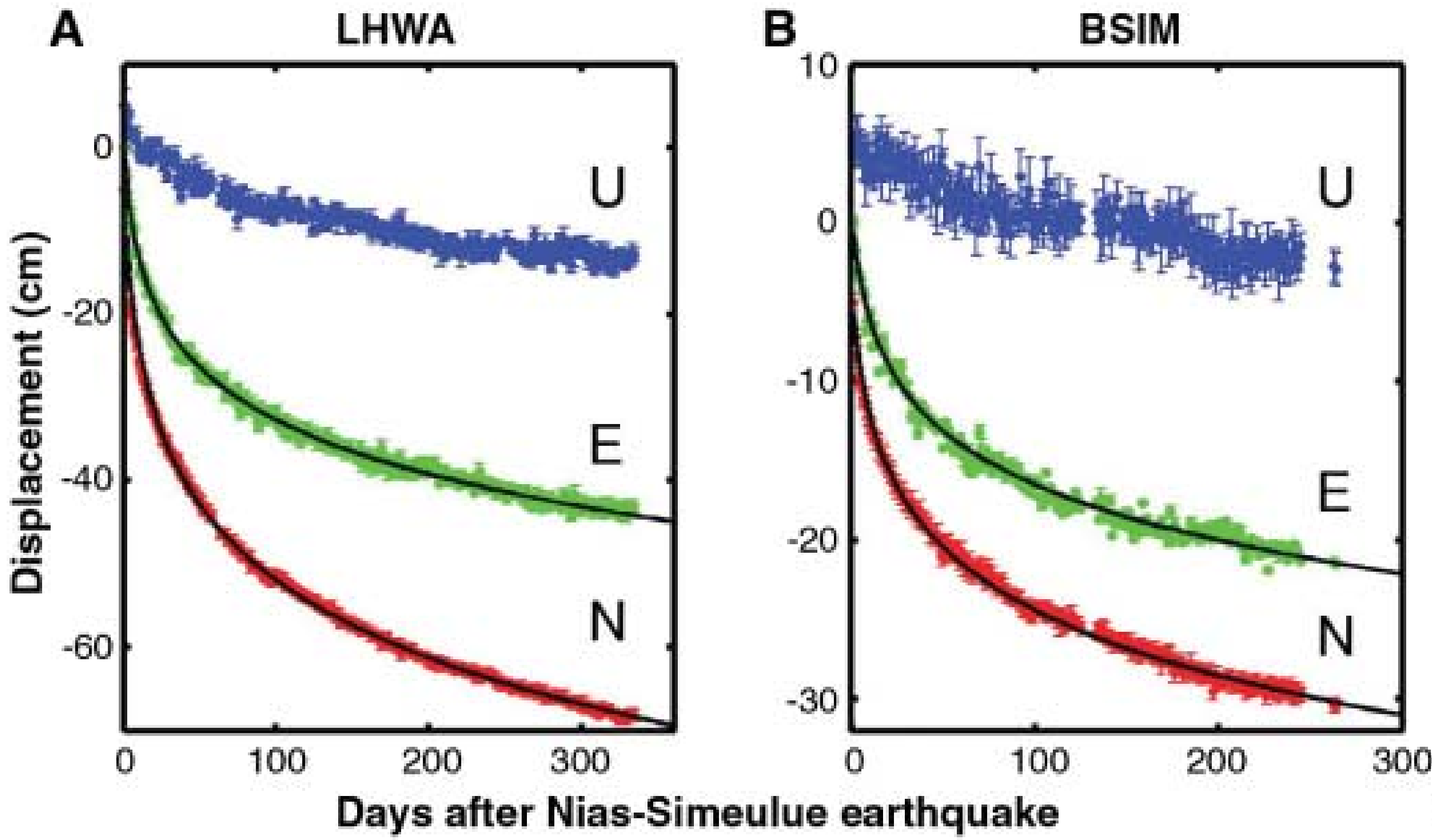
Characteristics of Afterslip

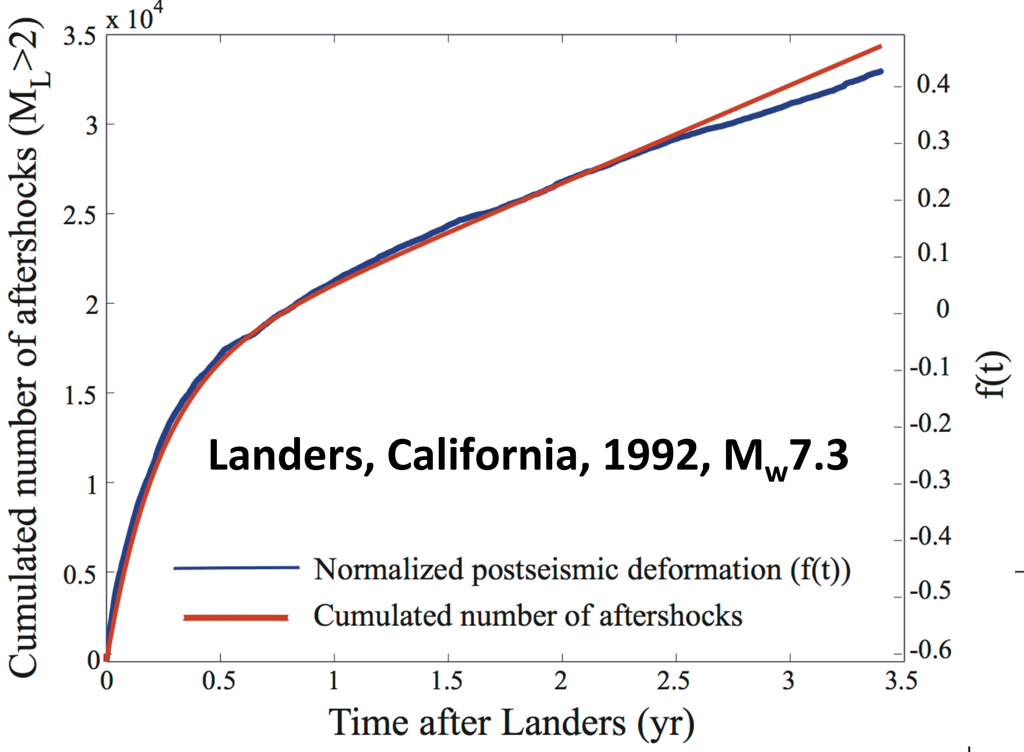
Afterslip of Nias (2005, M_w=8.7)



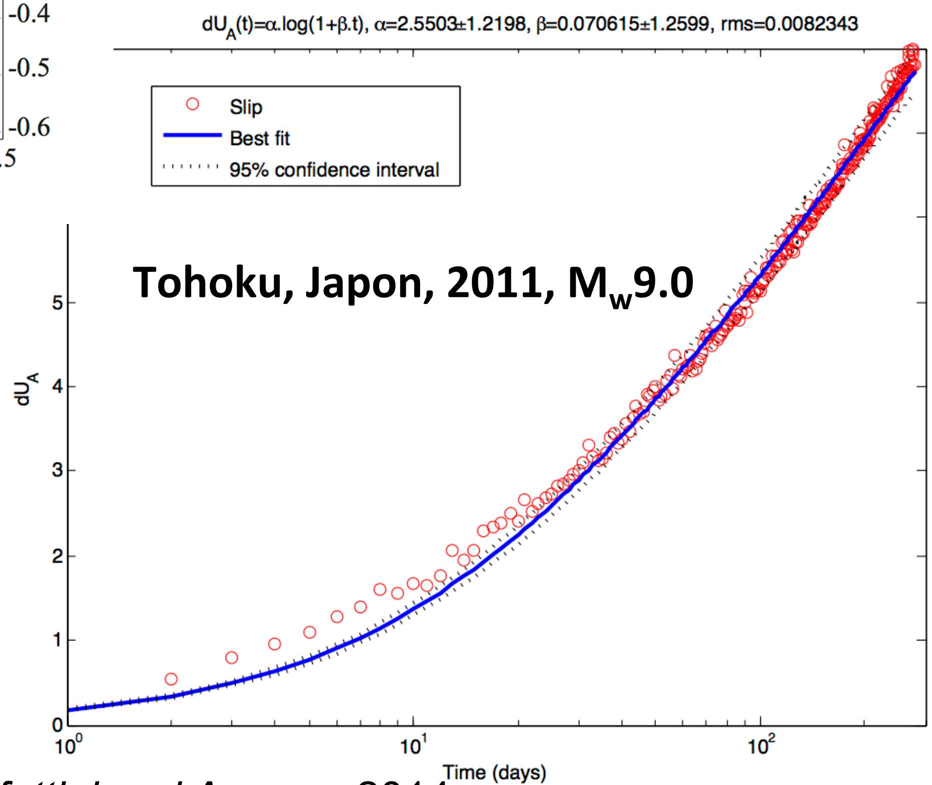
Hsu et al., 2006

Postseismic slip grows as the log(time)

Other Cases



Savage and Svarc,1997



Perfettini and Avouac, 2014

Afterslip is a rate strengthening process

demonstration adapted from *Avouac* [2015]

Assuming that surface displacement evolves as the logarithm of time and because the medium is elastic, afterslip on the fault is given by

$$U=U_0\times\log(1+t/t_0),\ U_0>0\ (1)$$

The frictional stress is related to fault slip by

$$\tau=\tau_0-KU,\ K>0\ (2)$$

where **K** is the stiffness of the fault.

Differentiating (1) with respect to time gives

$$V=dU/dt=V_0/(1+t/t_0)\ (3)$$

with

$$V_0=U_0/t_0$$

Combining Eqs. (1) with (2) gives

$$\tau=\tau_0 - KU_0 \times \log(1+t/t_0) \quad (4)$$

Using (3), (4) becomes

$$\tau=\tau_0 + K\,U_0 \times \log(V/V_0) \quad (5)$$

The growth of surface displacement with the logarithm of time (Eq. (1)) and the presence of an elastic medium (Eq. (2)) surrounding a slipping region of fixed sized (for K to be defined) **implies that the frictional stress is rate strengthening** and that **the functional dependence is logarithmic** (so highly non-linear) as observed in laboratory experiment..

Afterslip Models

Assuming steady-state rate and state friction

$$\tau=\tau_*+A\log(V/V_*)$$

where

$$A=\sigma(a-b)>0$$

σ: normal stress

a, b: rate and state parameters

Marone et al. (1991) proposed the following form for afterslip

$$U=(A/K)\log(1+t/t_0)+V_L t$$

$$t_0=A/(KV_{cs})$$

V_{cs}: coseismic slip velocity

V_L: loading or long term velocity

K: equivalent stiffness of the afterslip region

The formula

$$U=(A/K)\log(1+t/t_0)+V_L t$$

predicts

- A logarithmic growth of postseismic slip as the logarithm of time, consistent with the observations
- An infinite amount of afterslip at large time and no return to a steady-state velocity
- The characteristic time $\mathbf{t_0=A/(KV_{cs})}$ is mainshock dependent throught $\mathbf{V_{cs}}$

Perfettini and Avouac (2004) derived a modified expression given by

$$\mathbf{U=V_L t_r \log[1+(V_+/V_L)\times(\exp(t/t_r)-1)]}$$

where

$$\mathbf{t_r=A/(KV_L)}$$

$\mathbf{t_r}$: duration of the postseismic phase

$$\mathbf{V_+/V_L=\exp(\Delta CFS/A)}$$

ΔCFS: Coulomb stress change induced by the mainshock in the afterslip region

The formula $\mathbf{U=V_L t_r \log[1+(V_+/V_L)\times(\exp(t/t_r)-1)]}$ implies

- A logarithmic growth of postseismic slip as the logarithm of time, consistent with the observations

- When $\mathbf{t \gg t_r}$, this expression implies that

$$\mathbf{U \sim V_L t}$$

And the long term sliding velocity of the creeping region is the loading velocity

- The characteristic time $\mathbf{t_r=A/(KV_L)}$ is **not mainshock dependent**

- $\mathbf{V_+}$ is the sliding velocity of the creeping region right after the mainshock

In the limit $t \ll t_r$, $U = V_L t_r \log[1+(V_+/V_L)\times(\exp(t/t_r)-1)]$ reduces to

$$U \sim V_L t_r \log[1+t/t_1],\ t \ll t_r \quad \text{(I)}$$

where

$$t_1 = (V_L/V_+) t_r = A/(KV_+)$$

Since $V_L t_r = A/K$ and $t_0 = A/(KV_{cs})$, Eq. (I) reduces to

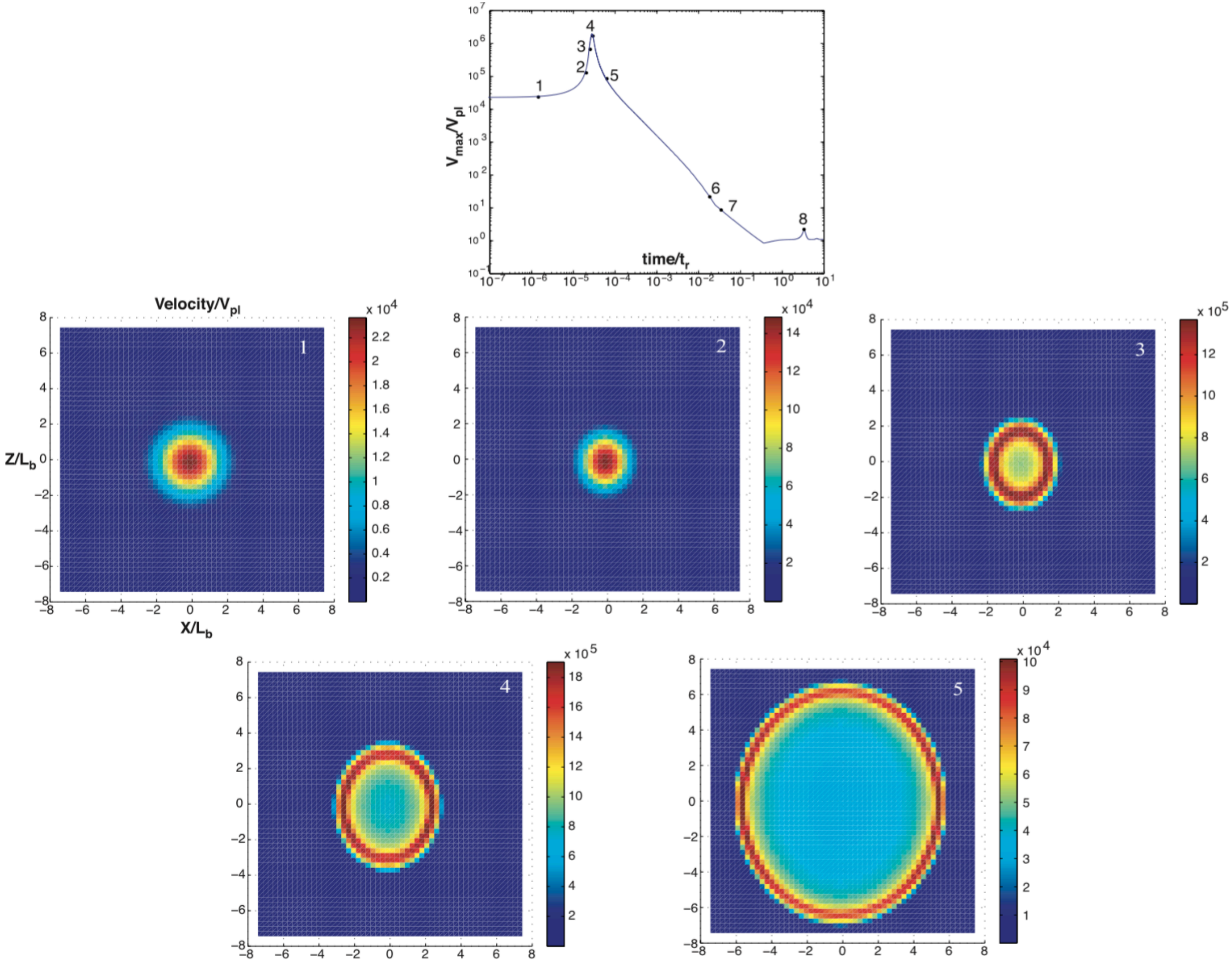
$$U = (A/K) \log(1+t/t_0) \quad \text{(II)}$$

assuming $V_{cs} = V_+$ so that $t_1 = t_0$.

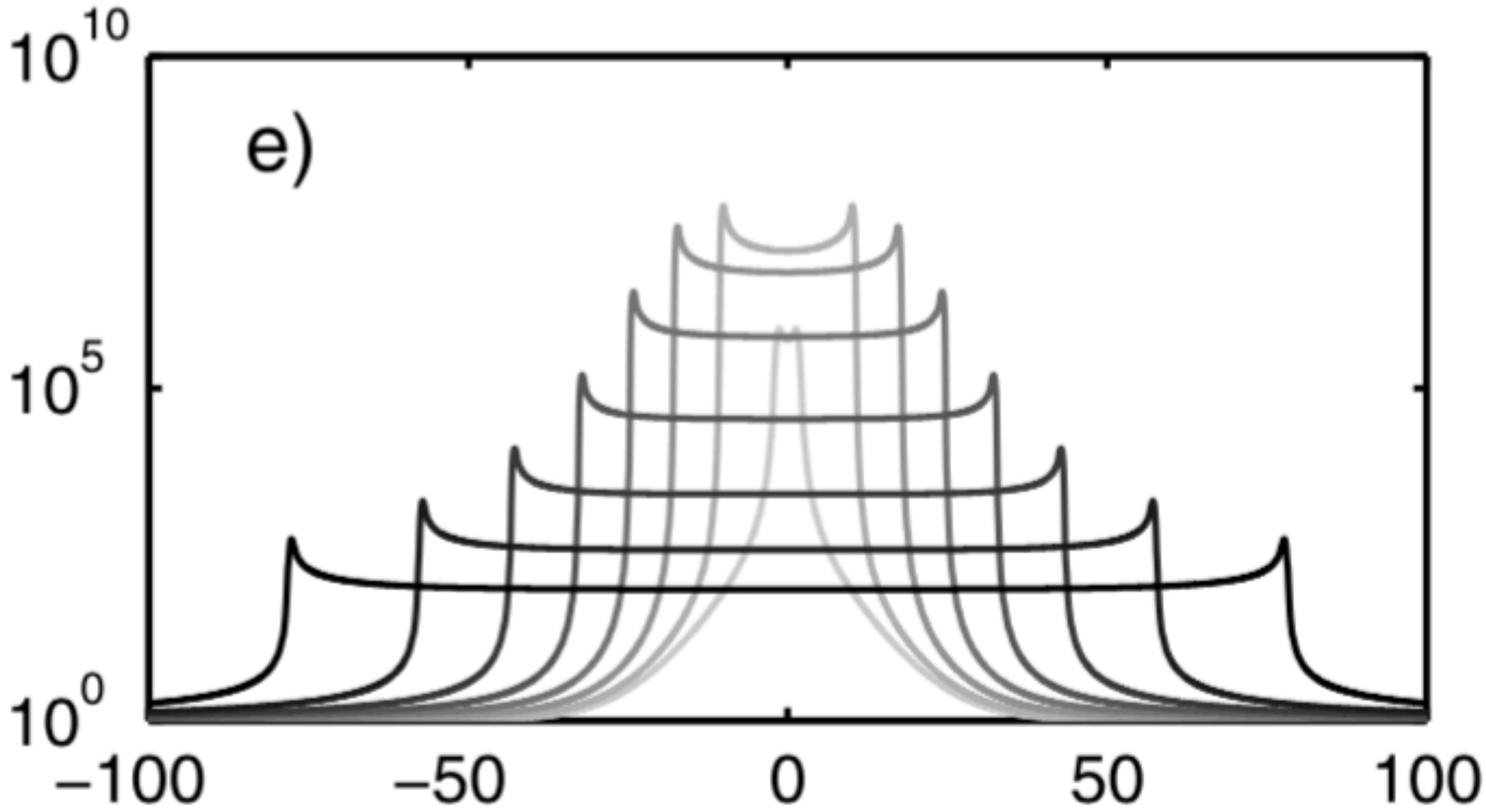
So the approach of *Perfettini and Avouac* (2004) reduces to the one of *Marone et al.* (1991) in the limit of small observation time (**$t \ll t_r$**).

The characteristic time **$t_r = A/(KV_L)$** is independent of the mainshock and is related to the frictional properties (through **A**) and size (through **K**) of the creeping region, and on the loading (or long term) sliding velocity of the creeping region.

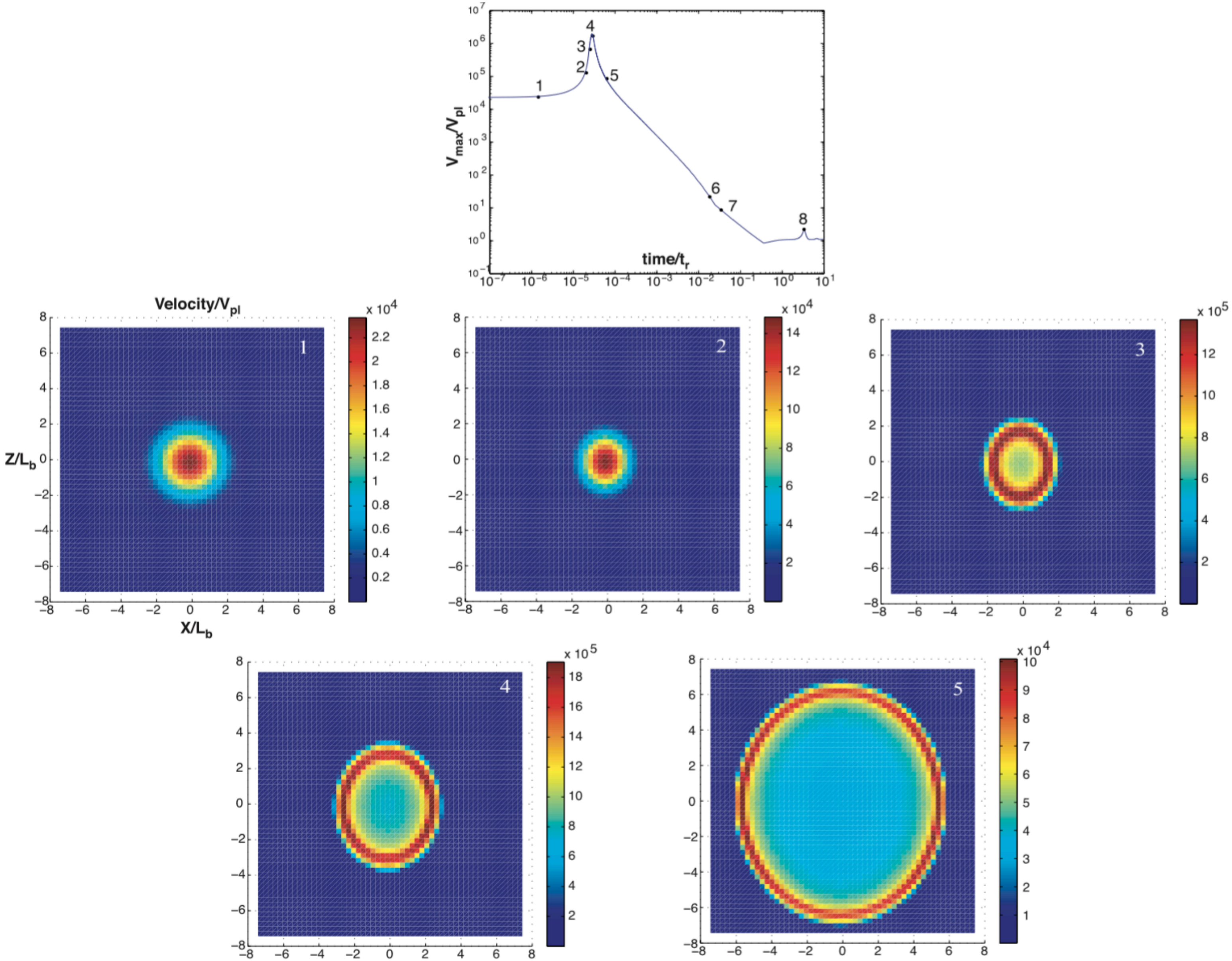
Validity of the Steady-State Approximation



Perfettini and Ampuero (2008) showed that following a stress step, the rate strengthening region starts to accelerate



- An initial accelerating phase during which slip localize over a region of size $\mathbf{L_b}$
- The acceleration phase is followed by a relaxation phase where slip spreads in a crack-like manner
- The size of the acceleration region is $\mathbf{L_b = G\ d_c / b\sigma}$, similar to the nucleation of a rate weakening fault



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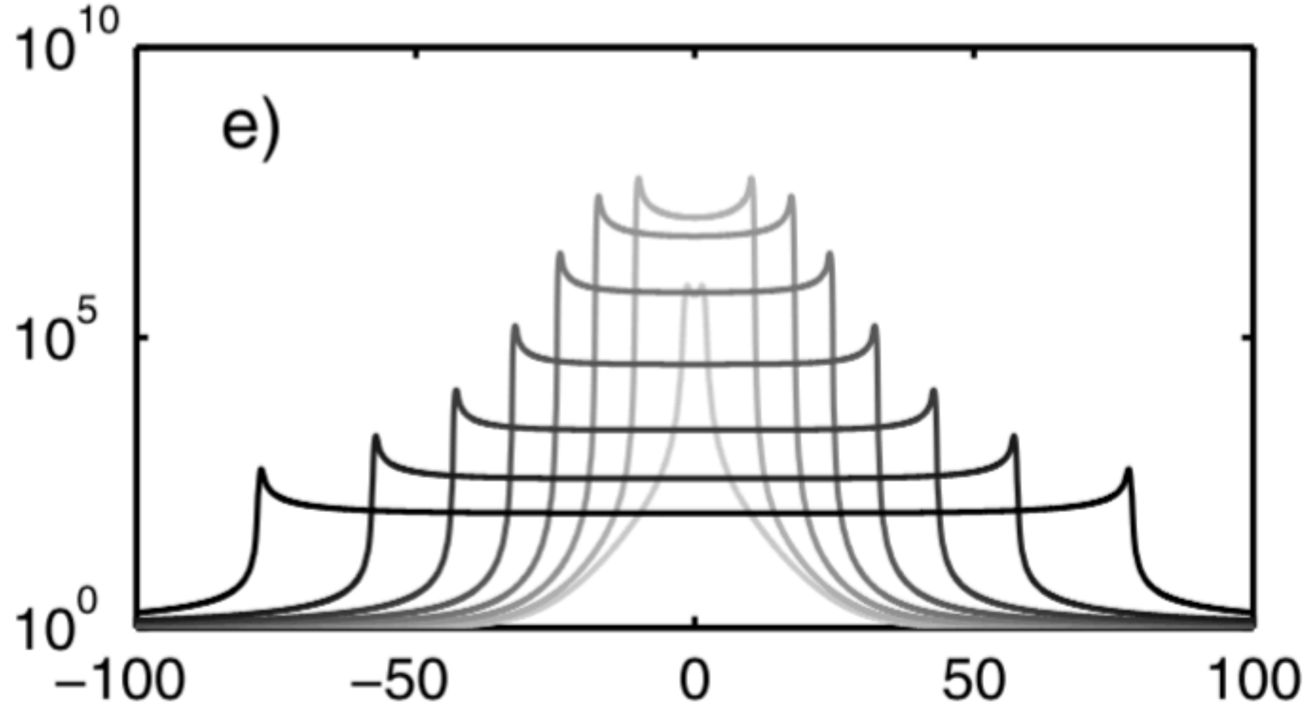


- If the size of the initial stress perturbation is **larger** than the nucleation size $\mathbf{L_b = G\ d_c/b\sigma}$, there is an **initial acceleration phase**

- If the size of the initial stress perturbation is **smaller** than the nucleation size $\mathbf{L_b = G\ d_c/b\sigma}$, there is **no initial acceleration phase** and the steady state approximation is always valid

- When existing, The duration of the acceleration phase is $\mathbf{t_{max} \sim (a/b)\, d_c / V_+}$ where $\mathbf{V_+ = V_L \exp(\Delta\tau / a\sigma)}$, after which relaxation occurs at steady-state

- Based on laboratory values, the duration of $\mathbf{t_{max}}$ spans an enormous range of short time scales, from 10^{-6} s to a few days (depending on the value of $\mathbf{d_c}$)

- The maximum velocity is $\mathbf{V_{max} \sim V_L \exp[\Delta\tau / (a-b)\sigma]}$ as it would be assuming the steady-state approximation

- So deviations from steady-state, if existing, should be noticeable only in the very early stage of the postseismic phase

Physical Basis of the Rate Strengthening Rheology

Eyring transition-state theory [*Eyring*, 1935] to determine the frequency ν with which an event occurs when it has to overcome a potential energy barrier of height E_a

$$\nu=\nu_0 \exp[-(E_a-\tau\Omega)/(k_B T)]$$

τ: applied stress
ν_0: reference frequency
Ω: activation volume
(volume activated by the process)
T: temperature
k_B: Boltzmann constant

Under the presence of body deformation, the height of the energy barrier is reduced by an amount $\tau\Omega$.

Probability $\mathbf{P_+}$ for the activated volume to move in the direction of the applied stress

$$P_+=P_0 \exp[-E_a/(k_B T)] \exp[\tau\Omega/(k_B T)]$$

Probability $\mathbf{P_-}$ for the activated volume to move in the direction opposite to the applied stress

$$P_-=P_0 \exp[-E_a/(k_B T)] \exp[-\tau\Omega/(k_B T)]$$

The mean sliding velocity is given by

$$V=V_0 [P_+ + P_-]$$

$\mathbf{V_0}$: reference velocity

$$V = V_* \{\exp[\tau\Omega/(k_B T)] + \exp[-\tau\Omega/(k_B T)]\}$$

with

$$V_* = V_0 P_0 \exp[-E_a/(k_B T)]$$

yielding

$$V = 2V_* \sinh[\tau\Omega/(k_B T)]$$

To relate $\boldsymbol{\tau}$ to **V**, we assume that the probability of a backward jump is negligible ($\mathbf{P_+ >> P_-}$)

$$V \approx V_* \exp[\tau\Omega/(k_B T)]$$

Since

$$V \approx V_* \exp[\tau\Omega/(k_B T)]$$

we get

$$\tau \approx A \log(V/V_*)$$

with

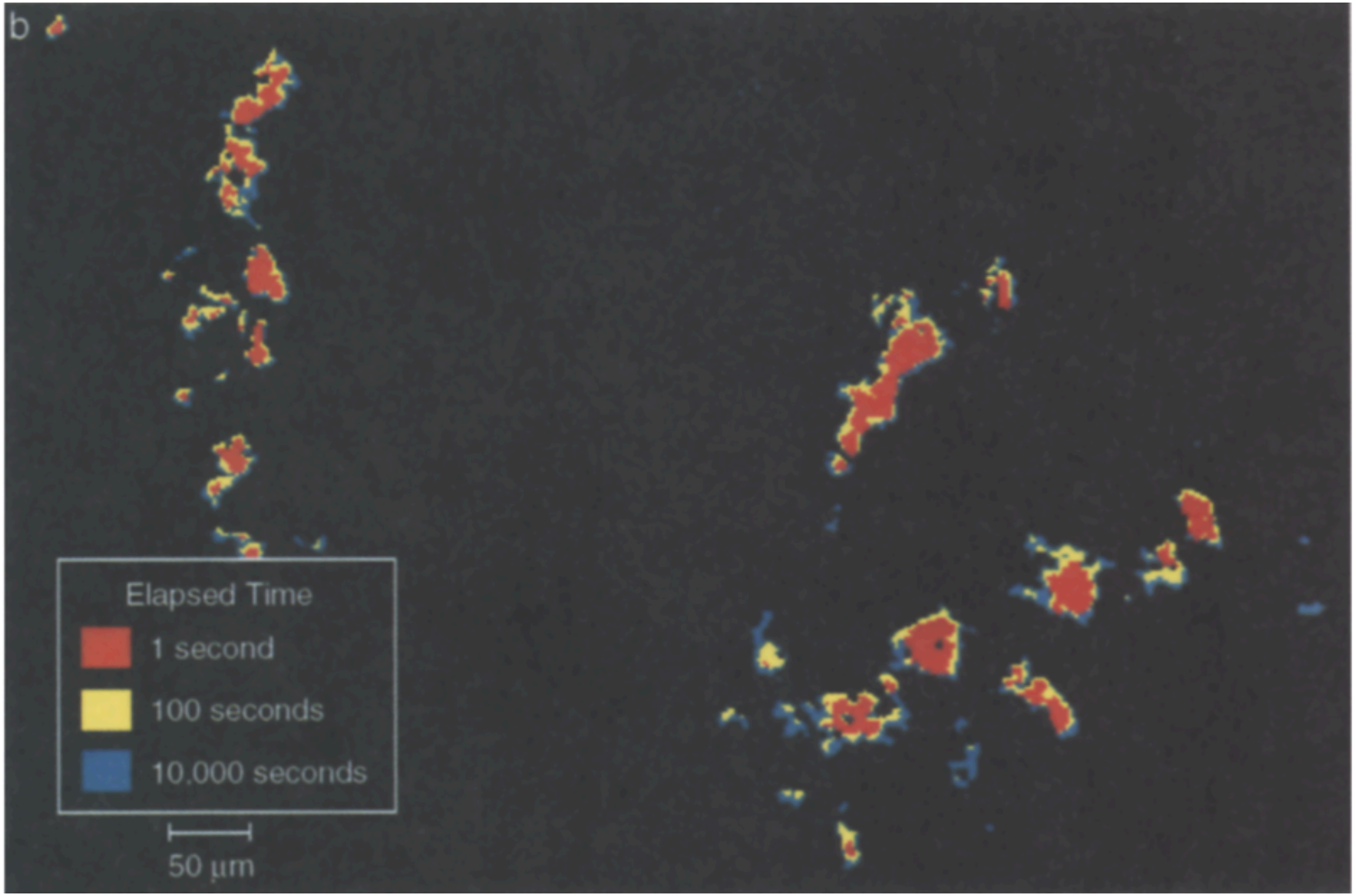
$$A = (k_B T/\Omega) > 0$$

So thermally activated processes (dislocation creep, diffusion creep, pressure solution creep…) lead to a logarithmic rate strengthening rheology.

The rheological parameter increases linearly with **T**.

Rate and state friction and the Bowden-Tabor theory of friction

Because surfaces are rough, the frictional contact is sustained by a small fraction (typically 0.1-1 %) of the nominal contact area.



Dieterich and Kilgore, 1996

Therefore, the few contacts bearing the contact support a huge load, beyond their elastic limit.

In the Bowden-Tabor theory of friction, it is assumed that those contacts are plastic and that the frictional force is given by

$$\mathbf{F = \tau_p \Sigma_r}$$

F: frictional force
τ_p: plastic stress
Σ_r: real contact area

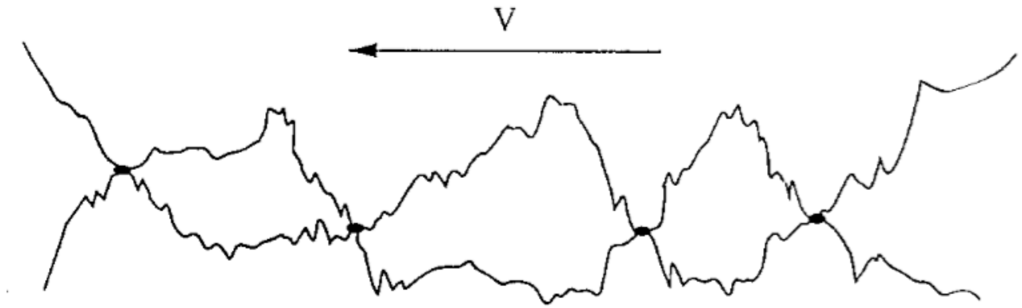


Figure 1
Geometry of asperity junctions (schematic).

Estrin and Bréchet, 1996

The frictional stress is given by

$$\tau = \tau_p \Sigma_r / \Sigma_0$$

τ: frictional stress
Σ_0: total (nominal) contact area

We study small deviations from a reference state (…)*

$$\tau_p = \tau_p^* + \Delta\tau_p, \ |\Delta\tau_p| << \tau_p^*$$
$$\Sigma_r = \Sigma_r^* + \Delta\Sigma_r, \ |\Delta\Sigma_r| << \Sigma_r^*$$

All parameters further derived will depend on the value of the reference state.

$$\tau = (\tau_p^* + \Delta\tau_p)\times(\Sigma_r^* + \Delta\Sigma_r)/\Sigma_0$$

$$\tau = (\tau_p^*\Sigma_r^* + \tau_p^*\Delta\Sigma_r + \Delta\tau_p\Sigma_r^* + \Delta\tau_p\,\Delta\Sigma_r)/\Sigma_0$$

The term $\Delta\tau_p\,\Delta\Sigma_r$ is a 2nd order term and is negligible (not true if the perturbation is not infinitesimal)

$$\tau = (\tau_p^*\Sigma_r^* + \tau_p^*\Delta\Sigma_r + \Delta\tau_p\Sigma_r^*)/\Sigma_0$$

It is assumed that under steady-state sliding (Arrhenius type law or Eyring theory)

$$\Delta\tau_p^{ss} = \tau_p^*\,\beta_\tau \log(V/V_*)$$

$\beta_\tau \ll 1$: material constant

τ_p^*: plastic stress at the reference state

V_*: sliding velocity in the reference state

$$\tau^{ss} = [\tau_p^* \Sigma_r^* + \Sigma_r^* \tau_p^* \beta_\tau \log(V/V_*) + \tau_p^* \Delta\Sigma_r^{ss}]/\Sigma_0$$

τ_p^*: frictional stress under steady-state sliding

The friction coefficient $\mu_{ss}=\tau/\sigma_*$ in steady state sliding is given by

$$\mu_{ss} = \mu^* + a \log(V/V^*) + \tau_p^* \Delta\Sigma_r^{ss}/(\sigma_*\Sigma_0)$$

σ_*: normal stress in the reference state

with

$$\mu^* = (\tau_p^*/\sigma_*)\times(\Sigma_r^*/\Sigma_0)$$

$$a = (\tau_p^*/\sigma_*)\times(\Sigma_r^*/\Sigma_0)\beta_\tau = \beta_\tau \mu^*$$

It is assumed that under steady-state sliding

$$\Delta\Sigma_r^{ss} = -\Sigma_r^* \beta_\Sigma \log(V/V_*)$$

β_Σ**<<1**: material constant

Finally, we get

$$\mu_{ss} = \mu^* + (a-b) \log(V/V^*)$$

with

$$b = (\tau_p^*/\sigma_*) \times (\Sigma_r^*/\Sigma_0)\beta_\Sigma = \beta_\Sigma \mu^*$$

We have related the μ^*, **a**, and **b** parameters to the elementary parameters of the model.

Note that since $a = \beta_\tau \mu^*$, stability of frictional sliding is controlled by the parameter

$$b/a = \beta_\Sigma/\beta_\tau$$

We will assume that the following equation

$$\mu_{ss} = \mu^{*} + a \log(V/V^{*}) + \tau_p^{*} \Delta\Sigma_r^{ss}/(\sigma_{*}\Sigma_0)$$

is valid away from steady-state so that

$$\mu = \mu^{*} + a \log(V/V^{*}) + \tau_p^{*} \Delta\Sigma_r/(\sigma_{*}\Sigma_0)$$

Since $\mathbf{b = (\Sigma_r^{*}\tau_p^{*})/(\sigma_{*}\Sigma_0)\ \beta_\Sigma}$, the friction coefficient can be written as

$$\mu = \mu^{*} + a \log(V/V^{*}) + b\ [\Delta\Sigma_r/(\beta_\Sigma\, \Sigma_r^{*})]$$

Comparing

$$\mu = \mu^{*} + a \log(V/V^{*}) + b\,[\Delta\Sigma_r/(\beta_\Sigma \Sigma_r^{*})]$$

with

$$\mu = \mu^{*} + a \log(V/V^{*}) + b \log(\theta/\theta_{*})$$

gives

$$\theta = \theta_{*} \exp[\Delta\Sigma_r/(\beta_\Sigma \Sigma_r^{*})]$$

The state variable is directly related to the changes of contact area

Since

$$\mu = \mu^* + a \log(V/V^*) + \tau_p^* \Delta\Sigma_r/(\sigma_* \Sigma_0)$$

it is recommended to use the form

$$\mu = \mu^* + a \log(V/V^*) + \Phi$$

as in this case

$$\Phi = \tau_p^* \Delta\Sigma_r/(\sigma_* \Sigma_0)$$

Φ is directly proportional to the changes of real contact area

- The Bowden-Tabor theory of friction support the rate and state framework

- To determine the evolution of friction, an evolution law relating the state variable to the variable of the system (slip, sliding velocity, normal stress, temperature…) is required.

- The existing evolution laws were **empirically** derived.

The most popular evolution laws are the aging law

$$d\theta/dt = 1 - V\theta/d_c$$

and the slip law

$$d\theta/dt = (-V\theta/d_c^{'})\,\log(V\theta/d_c^{'})$$

$\mathbf{d_c}$ and $\mathbf{d_c^{'}}$ are derived using laboratory data and are "evolution law" dependent.

Therefore, $\mathbf{d_c \neq d_c^{'}}$

During static contact (**$V \approx 0$**), the aging law becomes

$$d\theta/dt \approx 1$$

$$\theta \approx t + \theta(0)$$

$\theta(0)$: value of the state variable at $t=0$

which is the main reason for the physical interpretation of θ as representing the age of the contacts.

During static contact and considering the aging law, the changes of friction coefficient are given by

$$\Delta\mu = b \log[\theta(t)/\theta_0] = b \log[1 + t/\theta_0] = \Delta\Phi$$

It is observed experimentally on transparent materials that during static contact, the surface area grows logarithmically with time [*Dieterich and Kilgore*, 1996]. This phenomenon is responsible for **frictional healing**.

Since during stationary contact and under the aging law

$$\Delta\Phi = b \log[1 + t/\theta_0]$$

and given that $\Phi = \tau_p^* \Delta\Sigma_r/(\sigma_* \Sigma_0)$, the logarithmic increase of Φ with time is consistent with an increase of the plastic contact area $\Delta\Sigma_r$ with time.

Consequently, the interpretation of the state variable as the contact time is only true during stationary contact and considering the aging law.

Assuming that friction obeys the Bowden-Tabor theory, the proper physical meaning of the state variable is that it is related to the changes of contact area, not the contact time.

During static contact ($V \approx 0$), the slip law implies that

$$\mathbf{d\theta/dt \approx 0}$$

as $d\theta/dt=(-V\theta/d_c')\log(V\theta/d_c')\approx 0$ when $V\theta/d_c'\approx 0$

Therefore, the slip law predicts no evolution of the state variable (and hence the contact area) during static contact, contradicting laboratory experiments of frictional healing.

But the slip law well adjust velocity steps in the laboratory [*Bhattacharya et al.*, 2015] while the aging law does not.

None of the popular evolution laws are able to adjust the entire spectrum of laboratory experiments.

URGENT NEED FOR NEW EVOLUTION LAWS!!!

CONCLUSIONS

The estimate of the frictional parameters (μ_*, a, b,d_c) are:

- Material dependent
- Evolution law dependent
- Reference state dependent

Therefore, it appears highly risky to compare the values of the frictional parameters obtained considering different experimental setups… Something that is currently done.

What is the relevant rheology to model aseismic slip (for instance afterslip)?

- Rate strengthening rheology (dislocation creep, diffusion creep, pressure solution creep…) ?
- Rate and state friction under steady-state sliding ?
- Full rate and state friction ?

- Under the Bowden-Tabor theory of friction, the state variable is a proxy for the relative changes of real contact area

- The existing evolution laws (aging and slip) are empirically derived

- None of the existing evolution laws properly adjust simultaneously velocity steps and frictional healing experiments

- Need for new evolution laws

THANK YOU